2.1.1 $V_s = V_b + V_f \qquad V_d = V_b - V_f$

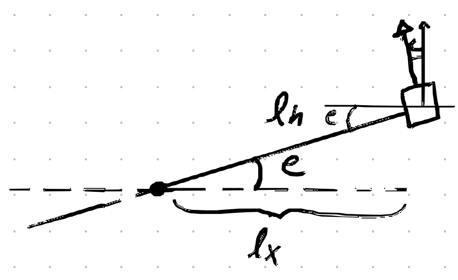
a) $J_e \ddot{\theta} = \Sigma \tau = \Sigma F \cdot r$

$\Rightarrow J_p \ddot{p} = -l_p\,(F_f - \cancel{F_{g,f}}) + l_p (F_b - \cancel{F_{g,b}})$

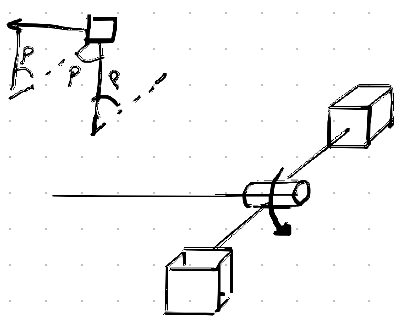
$= l_p (F_f + F_b)$

$J\ddot{p} = \cancel{l_p} K_F (V_b - V_f) = L_1 V_d$

b)



$\cos(e) = \frac{l_x}{l_h}$



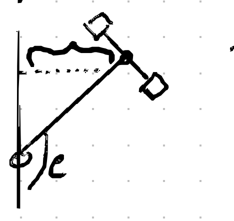
b) $J_e \ddot{e} = \Sigma F \cdot r = F_{g,c} \cdot l_c \cos e - (F_{g,f} + F_{g,b})\, l_h \cos e$

$+ (F_f + F_b)\, l_h \cos p$

$= \cos e\,(F_{g,c}\, l_c - (F_{g,f} + F_{g,b})\, l_h) + \cos p\,(F_f + F_b)\, l_h$

$= L_2 \cos e + L_3 V_s \cos p$

c) $J_\lambda \ddot{\lambda} = (F_f + F_b) \cdot (l_h \sin p \cos e) = L_4 V_s \cos e \sin p$



The larger e is, the less the propellers' "arm" is which can apply torque.

2.1.2

a) $J_p \ddot{p} = L_1 V_d \Rightarrow \ddot{p} = \frac{L_1}{J_p} V_d = K_1 V_d$

b) $J_e \ddot{e} = L_2 \cos e + L_3 V_s \cos p \qquad \frac{\partial}{\partial e}\cos e = -\sin e \qquad \frac{\partial}{\partial e}\cos p = 0$

$\approx L_2 + L_3 V_s + 0 + 0$

$\ddot{e} \approx \frac{L_2}{J_e} + \frac{L_3}{J_e}(\tilde{V}_s + V_{s,0})$

Equilibrium: $\ddot{e} = 0,\ \tilde{V}_s = 0$

$\Rightarrow 0 = \frac{L_2}{J_e} + \frac{L_3}{J_e}(0 + V_{s,0})$

$V_{s,0} = -\frac{L_2}{J_e} \cdot \frac{J_e}{L_3} = -\frac{L_2}{L_3}$

$\Rightarrow \ddot{e} = \cancel{\frac{L_2}{J_e}} + \frac{L_3}{J_e}\tilde{V}_s - \cancel{\frac{L_3}{J_e} \frac{L_2}{L_3}}$

$= \frac{L_3}{J_e}\tilde{V}_s = K_2 \tilde{V}_s$

c) $\ddot{\lambda} = \underbrace{\frac{1}{J_\lambda} L_4 V_s \sin p \cos e}_{f_3(V_s, e, p)} \qquad \ddot{\lambda} = f(x_0) + \nabla f(x_0) \cdot (x - x_0)$

$\nabla f_3 = \frac{1}{J_\lambda}\begin{bmatrix} L_4 \sin p \cos e \\ -L_4 V_s \sin p \sin e \\ L_4 V_s \cos p \cos e \end{bmatrix}$ evaluate i $\begin{matrix} p = 0 \\ e = 0 \\ V_s = V_{s,0} \\ V_d = 0 \end{matrix} \Rightarrow \frac{1}{J_\lambda}\begin{bmatrix} 0 \\ 0 \\ L_4 V_{s,0} \end{bmatrix}$

$\Rightarrow \ddot{\lambda} = \nabla f_3^T \Big|_{\substack{p=0 \\ e=0 \\ V_s = V_{s,0}}} \begin{bmatrix} V_s - V_{s,0} \\ e - 0 \\ p - 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & L_4 V_{s,0} \end{bmatrix} \begin{bmatrix} V_s - V_{s,0} \\ e - 0 \\ p - 0 \end{bmatrix}$

$= \frac{1}{J_\lambda} L_4 V_{s,0}\, (p) = K_3 p$

2.1.3

a) $V_d = K_{pp}(p_c - p) - K_{pd}\dot{p}$

$\hookrightarrow \ddot{p} = K_1 (K_{pp}(p_c - p) - K_{pd}\dot{p})$

$\ddot{p} + K_1 K_{pd}\dot{p} + K_1 K_{pp} p = K_1 K_{pp} p_c$

$\hat{p}s^2 + K_1 K_{pd}\hat{p}s + K_1 K_{pp}\hat{p} = K_1 K_{pp}\hat{p}_c$

$\frac{\hat{p}}{\hat{p}_c} = \frac{K_1 K_{pp}}{s^2 + K_1 K_{pd} s + K_1 K_{pp}}$

2.1.4

a) $D_{et} = s^2 + K_1 K_{pd} s + K_1 K_{pp} = 0 \Rightarrow S_{1,2} = \frac{-K_1 K_{pd} \pm \sqrt{(K_1 K_{pd})^2 - 4 \cdot 1 \cdot K_1 K_{pp}}}{2}$

Assume $D_{et} = (s - \lambda_1)(s - \lambda_2) = s^2 - (\lambda_1 + \lambda_2)s + \lambda_1 \lambda_2$

$\Rightarrow -(\lambda_1 + \lambda_2) = K_1 K_{pd}$

$\lambda_1 \lambda_2 = K_1 K_{pp}$

$\Rightarrow K_{pp} = \frac{\lambda_1 \lambda_2}{K_1}$

$K_{pd} = -\frac{(\lambda_1 + \lambda_2)}{K_1}$

Test plan

$\lambda_1\,\lambda_2$		$\lambda_1\,\lambda_2$	
	✓		✓
	✓		✓
	✓		✓
	✓		✓
	✓		

$L_1 = l_p K_F \qquad L_3 = l_h (F_f + F_b) = l_h (V_f K_f + V_b K_f) = K_f l_h$

$L_2 = F_{g,c} l_c - (F_{g,f} + F_{g,b}) l_h$

$V_{s,0} = -\frac{L_2}{L_3} \qquad V_{s,0} = -\frac{F_{g,c} l_c - (F_{g,f} + F_{g,b}) l_h}{K_f l_h (V_f + V_b)}$

$K_f = -\frac{F_{g,c} l_c - (F_{g,f} + F_{g,b}) l_h}{V_{s,0}\, l_h}$

$K_1 = \frac{L_1}{J_p} = \frac{l_p K_F}{J_p} = -\frac{l_p}{J_p} \frac{F_{g,c} l_c - (F_{g,f} + F_{g,b}) l_h}{V_{s,0}\, l_h}$

$K_{pp} = \frac{\lambda_1 \lambda_2}{K_1} \qquad K_{pd} = \frac{-(\lambda_1 + \lambda_2)}{K_1}$